AMERICAN UNIVERSITY OF BEIRUT
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

EECE 440	SIGNALS AND SYSTEMS	Spring 2004-2005
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Problem 1 (10 pts)

A system whose input-output relationship is given by:

$$y(t) = \int_{-\infty}^{+\infty} e^{-\tau} u(-\tau) x(t-\tau) d\tau$$

a. Is this system linear? Justify your answer (2 pts)
   Yes
b. Is this system time-invariant? Justify your answer (2 pts)
   Yes
c. Is this system memoryless? Justify your answer (2 pts)
   No, y(t) dependents on future values of x(t)
d. Is this system Causal? Justify your answer (2 pts)
   No, $h(t) = e^{-t}u(-t)$
e. Is this system Stable? Justify your answer. (2 pts)
   No, $h(t) \to \infty$ as $t \to -\infty$

Problem 2 (3 pts)

For the following statement, if you believe it is true, give a justification. If you believe it is false, give a counterexample.
"A linear, causal, and continuous-time system is always time-invariant".

Not true. Counterexample: y(t)=tx(t)

Problem 3 (3 pts)

For the following statement, if you believe it is true, give a justification. If you believe it is false, give a counterexample.

"The system with real-valued input x(t) and output given by: $y(t) = \left(1 + x^2(t)\right)^{\cos t}$ is stable."

True, if x(t) is bounded, y(t) is bounded

Problem 4 (3 pts)

For the following statement, if you believe it is true, give a justification. If you believe it is false, give a counterexample.

"The system with input x(t) and output given by: $\frac{dy(t)}{dt} + y(t) = x(t)$ is stable."

True; $H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$. No poles in the rhp or jw axis

Problem 5 (3 pts)

Determine whether or not the following signal is periodic. If the signal is periodic, determine its fundamental period.

$$x(t) = \cos(\frac{\pi}{3}t) + \sin(\frac{3\pi}{4}t - \pi)$$

$\cos(\frac{\pi}{3}t)$ is periodic with a period T1=6 s 1 pt

$\sin(\frac{3\pi}{4}t - \pi)$ is periodic with a period T2=8/3 s 1 pt

T=LCM(6, 8/3)=24 s 1 pt

Problem 6 (3 pts)

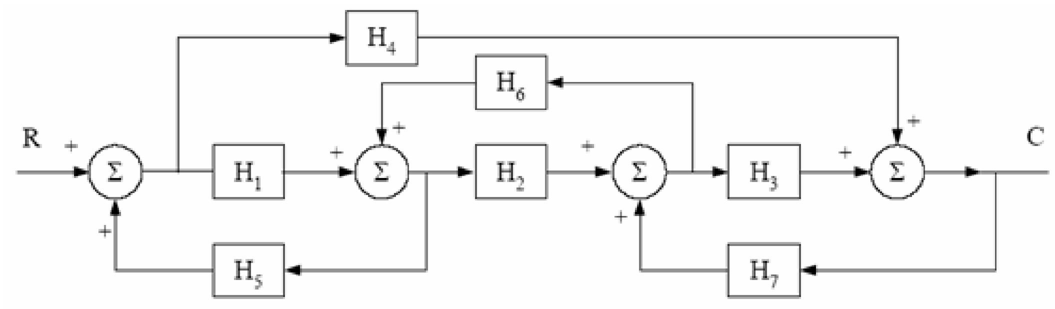
For the following statement, if you believe it is true, give a justification. If you believe it is false, give a counterexample.

"The signal $x(t) = \sin(t/1000)$ is a power signal".

x(t) is a periodic signal. Therefore, it is a power signal

Problem 7 (3 pts)

Find the transfer function of the following system.

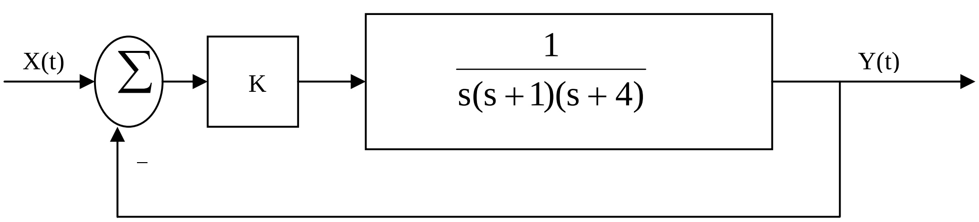


$$TF = \frac{H_1H_2H_3 + H_4[1 - H_2H_6]}{1 - [H_1H_5 + H_3H_7 + H_2H_6 + H_4H_5H_6H_7] + [H_1H_3H_5H_7]}$$

Grading policy: -1 for every incorrect or missing term

Problem 8 (6 pts)

Consider the unit feedback system shown below



a. Determine the error signal E(s). (3 pts)

$$E(s) = \frac{X(s)}{1 + G(s)} = \frac{X(s)[s(s+1)(s+4)]}{s(s+1)(s+4) + K}$$

b. Determine the range of K for the system to be stable (3 pts)

$$\frac{X(s)}{Y(s)} = \frac{K}{s^3 + 5s^2 + 4s + K}$$

RH table

S^4	1	5
S^3	5	K
S	$\frac{20-K}{5}$	
S^0	K	

For stability, K>0 and 20-K>0, implies 0<K<20

Problem 9 (4 pts)

Consider a single-tome modulated DSB-LC signal. The percentage modulation of this signal is 120%. Determine the lowest amplitude value of the envelope of this DSB-LC signal. The highest value of the envelope is assumed to be equal to 2 Volts.

For a single-tome modulation

$$s(t) = A_c[1+\mu\cos(\omega_m t)]\cos(\omega_c t)$$

Amplitude of s(t)= $A_c[1+\mu\cos(\omega_m t)]$

$A_{max} = A_c[1+\mu)] = 2 \Rightarrow A_c = 0.909$ Volts 2 pts

$A_{min} = A_c[1-\mu)] = -0.1818$ 1pt

Therefore, the lowest value of the envelope of s(t) is 0 V 1 pt

Problem 10 (4 pts)

The impulse response of a low-pass filter is h(t) and its transform is H(ω). If the transform of the output $Y(\omega) = 2H(\omega)e^{-j\omega}$, find the input x(t).

$H(\omega) = \frac{Y(\omega)}{X(\omega)} \Rightarrow X(\omega) = 2e^{-j\omega}$ 2pts

$h(t) = 2\delta(t-1)$ 2 pts

Problem 11 (8 pts)
A DSB-LC modulated wave has the form

$$s(t) = 10[1 + 0.5\cos(200\pi t) + 0.5\cos(400\pi t)]\cos(2000\pi t)$$

a. Determine the spectrum of s(t) (2 pts)

$$s(t) = 10\cos(2000\pi t) + 2.5\cos(2200\pi t) + 2.5\cos(1800\pi t) + 2.5\cos(2400\pi t) + 2.5\cos(1600\pi t)$$

for every cos(at), $S(\omega) = \pi[\delta(\omega - a) + \delta(\omega + a)]$

b. Find the total power of s(t) (2 pts)

$$P_t = \frac{(10)^2}{2} + 4\frac{(2.5)^2}{2} = 62.5 \text{ Watts}$$

c. Find the sideband power (2 pts)

$$P_s = 4\frac{(2.5)^2}{2} = 12.5 \text{ Watts}$$

d. Find the modulation index. (2 pts)

$$\frac{P_s}{P_t} = \frac{\mu^2}{2 + \mu^2} \Rightarrow \mu = 0.707$$

Problem 12 (4 pts)
Consider the DSB-LC signal

$$s(t) = A_c[1 + 0.5\cos(\omega_m t)\cos(2\omega_m t)]\cos(\omega_c t) \text{ with } \omega_c >> \omega_m.$$

Let s(t) be inputted to an ideal band-pass filter centred at ω_c rad/s and of bandwidth $4\omega_m \text{rad/s}$.

a. Determine the signal at the output of the filter. (2 pts)

$$v(t) = A_c[1 + 0.5\cos(\omega_m t)]\cos(\omega_c t)$$

b. Can an envelope detector be used to demodulate the signal determined in (a). If yes, determine the output of this envelope detector. (2 pts)

Yes, as the modulation index is less than 1. The output of the envelope detector is: $s(t) = A_c[1 + 0.5\cos(\omega_m t))]$

Problem 13 (6 pts)

Consider the signal

$$f(t) = \cos(\omega_c t) + 2j\sin(\omega_c t)$$

a. Determine the Hilbert transform of f(t) (2 pt)

$\hat{f}(t) = \sin(\omega_c t) - 2j\cos(\omega_c t)$

b. Determine the pre-envelope of f(t). (2 pt)

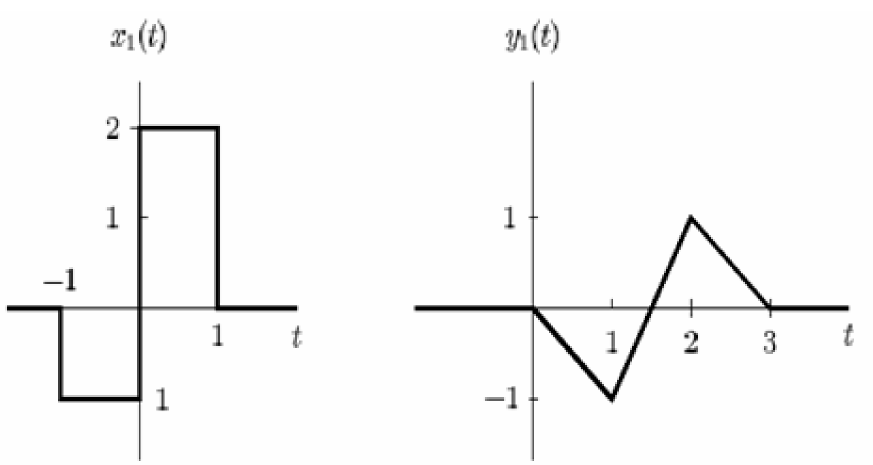
$f_+(t) = f(t) + j\hat{f}(t) = 3\cos(\omega_c t) + 3j\sin(\omega_c t) = 3e^{i\omega_c t}$

c. Determine the complex-envelope of f(t). (2 pt)

$\tilde{f}(t) = 3$

Problem 14 (5 pts)

The signal $x_1(t)$, shown below, is the input of an LTI system whose impulse response $y_1(t)$ is also shown below. Determine the output signal.



$$X_1(s) = \frac{1}{s}\left[-e^{s} + 3 - 2e^{-s}\right]$$ 1 pt

$$Y_1(s) = \frac{1}{s^2}\left[-1 + 3e^{-s} - 3e^{-2s} + e^{-3s}\right]$$ 1 pt

$$Y(s) = X_1(s)Y_1(s) = \frac{1}{s^3}\left[e^{s} - 6 + 14e^{-s} - 16e^{-2s} + 9e^{-3s} - 2e^{-4s}\right]$$ 1 pt

$$y(t) = \frac{1}{2}\left[\begin{array}{l}(t+1)^2u(t+1) - 6t^2u(t) + 14(t-1)^2u(t-1) - 16(t-2)^2u(t-2) \\ +9(t-3)^2u(t-3) - 2(t-4)^2u(t-4)\end{array}\right]$$